

Does private insurance erode the political support of social insurance?¹

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Abstract

What are the consequences of allowing private insurance as a supplement to social insurance? Is the scope of social insurance likely to be affected by an introduction of an optional supplemental private insurance?

The scope of social insurance is collectively decided and some may find that this is insufficient. This may give rise to the demand that it should be possible to get additional insurance in the private market. It is easy to conclude that as long as the private insurance is optional, introducing such an option harms no one. However, preferences towards social insurance are likely to be affected by the introduction. This may affect the scope of social insurance through political channels and this may harm other group than those choosing the private insurance.

An introduction of a private supplemental insurance may reduce the conflicts of interests and may allow for more individual freedom. It may also improve efficiency in the economy by reducing the public supply and thereby reducing the deadweight costs of taxation. However, a reduction in the public supply may reduce redistribution and equality in such an amount, that society is over all made worse off by the introduction of a private supplemental supply.

I examine under which conditions allowing for additional private insurance may be harmful for some groups. I find that this depends on the wage distribution, the deadweight costs of taxation and the conditions in the private insurance market.

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1 Introduction

The welfare state has been heavily debated in recent years. Should we roll back the spendings of the welfare state or not? Most of the focus has been on whether the welfare state improves or erodes overall efficiency in the economy.

"Growth and Employment: The Scope for a European Initiative" by Drèze and Malinvaud (1994) conclude that "the agenda should be to make the Welfare State leaner and more efficient" (p.82). "Turning Sweden around" (Lindbeck et al 1994) argues that the welfare state has "resulted in institutions and structures that today constitute an obstacle to economic efficiency and economic growth because of their lack of flexibility and their one-sided concerns for income safety and distribution, with limited concern for economic incentives" (p. 17). Martin Feldstein was concerned with the adverse effects of social security spending on economic performance. He concluded that "the social security program [in the United States] approximately halves the personal savings rate, [which] implies that is substantially reduces the stock of capital and the level of national income" (Feldstein 1974, p. 922)

These views are not the only ones however. Maddison (1984) stated that judgements of the influence of the welfare state on economic development were "influenced mainly by ideological positions, or predictions about what might happen in the future". Sandmo (1995) reviewed the aggregate empirical evidence between growth and social security and concluded that there was no clear connection either way.

A powerful defense of the welfare state is presented by Atkinson (1999). He concludes that "there can be little doubt about [the welfare state's] importance in providing income support". He finds no conclusive evidence between the aggregate relationships of economic performance and the size of the welfare state.

Efficiency is just one aspect of the discussion of the welfare state. Another is legitimacy. Rolling back the spendings of the welfare state may very well affect the legitimacy of the welfare state as well as the efficiency. In a democracy, the scope of the welfare state is not decided by economists, but by the population through political channels. A welfare state that is opposed by a large fraction of the population has a low degree of legitimacy and is not sustainable over time. A change in the welfare state which affects the majority's view of the welfare state, may thus have wider consequences than originally planned. When

political support of the welfare state is affected by the scope of the welfare state, there may exist a conflict between efficiency and legitimacy.

The welfare state may be a redistributive tool, with the less fortunate being subsidized by the more fortunate members of society. How efficient the welfare state is as such a tool is clearly dependent on its legitimacy.

Moene and Wallerstein (1999) investigate the effect of changes in the inequality in pre-tax and transfer income on the political support for welfare policy. In contrast to the findings of Romer (1975), Roberts (1977) and Meltzer and Richard (1981), they find that "The impact of increasing inequality on the political support for redistributed policies depends critically on the way in which benefits are targeted when targeting is exogenous." (p.26). They conclude that "When benefits are mostly targeted to those without earnings,....., greater inequality of income reduces support for redistributive policies." and that "Political support for benefits targeted to those without earnings goes down as inequality increases."

Casamatta, Cremer and Pestieau (1998) present a model with majority voting to examine the impact of the redistributive degree of the social insurance on the political sustainability of the welfare state. They find that there is a potential trade-off between efficiency costs and the political sustainability. They find that "it may be appropriate to adopt a system which is less redistributive than otherwise optimal, in order to ensure political support for an adequate level of coverage" (p. 27). They also examine the effect of allowing supplemental private insurance. They find that private insurance does undermine the political support for social insurance, but that this nevertheless may increase the welfare of the poor.

Casamatta, Cremer and Pestieau (1999) extend the model by allowing for two overlapping generations. They examine the effects of changes in the population structure, Pay As You Go-systems vs. Fully Funded-systems and take tax distortions into consideration.

Casamatta, Cremer and Pestieau (1998), (1999), Moene and Wallerstein (1999) and (2001) all examine the need for a degree of universalistic welfare policies in contrast to means-tested welfare policies. They all find that a degree of universalistic welfare policies may be needed for political support. M & W (2001) conclude that " a limited welfare state that pays benefits only to the poor may be politically unsustainable in the absence of altruistic voting." Casamatta, Cremer and Pestieau (1999) find that universalistic policies

are not desirable for the social planner. They find however that universalistic policies may be desirable when majority voting is taken into consideration due to the need for political support.

I present a model of how political support for welfare policies depend on whether there are private alternatives to the goods offered by the welfare state. The welfare policies here are insurance against income loss, with all welfare spendings being received by those without other income. The question asked is: What is the optimal amount of consumption of a private good, if this good is supplied by the government in an equal amount for all, and is tax financed? The trade-off is that the higher the tax, the higher the consumption of this good, but this means lower consumption of other goods as well. The answer is clearly dependent on preferences and income. In addition, it depends on whether there are private alternatives to the publicly supplied good that may be less expensive for some.

My focus is on the government as a collective supplier of a private good. I disregard other aspect of the government. The policy space then becomes one-dimensional. The only question is how large it should be, i.e. how high should the taxes be. Political competition may drive the level of the public supply towards the ideal tax rate for the median voter, i.e. the voter with the median ideal tax rate. If more than 50% of the population prefers a lower, or higher, tax rate, political competition may drive the tax rate up, or down. The higher the median ideal tax rate, i.e. the more people who prefer a high tax rate, the higher the realized tax rate and supply of the good in question.

The good in question could be any private good. I have chosen insurance as an example, but only minor changes are needed to capture the essence for other commodities, e.g. health care, pensions or education. The Von Neuman-Morgenstern utility function is a quasi-concave utility function, strictly quasi-concave in the presence of risk aversion, while the role of the coefficient of relative risk aversion is filled by the elasticity of substitution.

That the welfare state is a collective supplier implies that the scope of the welfare state, i.e. the amount of goods supplied, is collectively decided through political channels. Given that people are different, some may be dissatisfied with this amount. Some may find it too extensive, some may find it insufficient. I apply median voter theories for the analysis of the support of the welfare state.

I assume that people vote strictly according to self-interests. It is possible to allow for

some degree of altruism, as done in Moene and Wallerstein (1999).

The welfare state described here, is only a social insurance system, with the government as a non-profit insurance agency. The scope of the welfare state (i.e. the tax rate) is assumed to be decided through political competition. The decisive voter is the median voter and the realized tax rate is the one that the median voter favors. The median voter is not necessarily the median income earner. This is shown in section four. An introduction of a private alternative may create a means-against-ends situation.

What are the effects of allowing for an optional additional private insurance on the scope of social insurance? I use social insurance as the term for the insurance supplied by the government and private insurance as the term for insurance supplied by the private insurance company. It is easy to conclude that private insurance can never hurt someone as long as it is optional. This is not straightforward however. An introduction of a private insurance may very well affect the preferences towards the social insurance for some groups of the population, and thereby affect the scope of the social insurance as well. This may imply a welfare loss for other groups.

There are several major differences between having the good supplied by the government in the above mentioned manner or by a private firm:

- While the amount of goods is socially decided through a political process in the public scheme, the amount is individually decided in the private scheme. The former promotes equality, while the latter promotes individual choice.
- If the consumption of the publicly supplied good is equal for all, and thereby unrelated to the amount of tax paid, the public scheme works in a redistributive manner, with consumption being more costly to the ones paying high taxes. In the private scheme there are no such aspects, with everyone paying the same price.
- With a tax financed public supply, there might be some deadweight costs, which may increase with the supply. There are no deadweight costs associated with private supply, but there may be rents, or profits, and other costs.

I do not consider informational aspects. Blomquist and Christiansen (1995) investigate how public supply of a private good may weaken the information constraints of non-linear taxation. I comment only very briefly on information aspects, even though there are many insights to be found here.

There may be reasons for wanting a mix of a public and a private supply. We may want to limit the deadweight costs of taxation or allow for more individual freedom in the consumption of the good in question. However, an introduction of a private supplement to the social scheme may have effects on the preferences of the population and through this have effects on the amount of the public supply. This may have undesired equality and redistribution effects.

The private supply may be more desirable for some parts of the population than for other parts. If the amount of consumption is unrelated to tax payments in the collective scheme, the higher-than-average taxpayers are subsidizing the lower-than-average taxpayers in the public system. Thus it may be preferable for some parts of the population to reduce the public part of the mix. If individuals are voting in accordance with their self-interests, this may effect the realized mix of the supply scheme.

If there are redistribution goals to the public supply, the social scheme is more favorable to some. With no price discrimination, the price in the private market is identical for all. The private scheme is thus relatively favorable to some.

Additional private consumption for some only makes these individuals better off if the amount of public consumption remains unchanged. However the private supplement may affect the preferences about the amount of public consumption and lead to a change in the public sector through political processes. This may have consequences for the other consumers in either a negative or positive manner. If the consequences are positive or zero, the private supplement leads to a Pareto-improvement. Someone is made better off without anyone being made worse off. If the consequences are negative, there is a trade-off between individual and social welfare. The optimal mix of publicly and privately consumption then depends on the social weights put on the losses and gains of the different individuals. The object here is to analyze under which conditions the private supplement may lead to a loss for someone, and to highlight some factors that may influence a potential loss.

In the next section I present the basic model, with a homogenous population and no private alternative. Government spending is a tax-financed transfer to the unemployed. I then allow for a private supplemental insurance and analyze the effects of this. In section three I make the population heterogeneous in the way that the risk of being unemployed is heterogeneous. Translated to another good, this would be that preferences over the

various goods are heterogeneous. I again allow for a private insurance. In section four I keep risk homogeneous, but introduce heterogeneous income and analyze the situation with and without a private insurance. Section five adds social mobility to the situation of heterogeneous income, with and without the private alternative. Section seven concludes.

2 Basic model

As a benchmark I start with a model with a homogenous population. It is homogenous in three dimensions:

1. preferences
2. uncertainty
3. income

Homogeneous preferences are maintained throughout. Point 2 and 3 is relaxed later.

The individuals are facing a constant risk of losing their income $(1 - p)$, where $0 < p < 1$. The probability p is assumed to be the same for all individuals. When losing their income, they receive a tax-financed support from the government, unrelated to prior earnings. The support goes exclusively to those without other income. This simplifies the analysis but is not crucial for the results. Moene and Wallerstein (1999) have a more general model, where persons who are employed receive a share of the welfare spendings. The post tax consumption of a person currently working is given by

$$C_E = (1 - t)w, \tag{1}$$

where t is the marginal tax rate and w is the wage. The consumption of those without earnings is given by total tax income $T(t)$ divided by the share of the population without earnings, $(1 - p)$.

$$C_N = \frac{T(t)}{1 - p} \tag{2}$$

Total tax-income is given by

$$T(t) = \tau(t)pw \quad (3)$$

Here average income is pw . The function $\tau(t)$ represents tax-income as a function of the tax rate, which implicitly incorporates the deadweight cost of taxation. Without deadweight cost, we have $\tau(t) = t$. With deadweight cost $\tau(t)$ is a strictly concave function with $\tau(0) = \tau(1) = 0$ and $\tau'(0) = 1$. A concave function implies increasing deadweight costs of taxation. All interesting values of t makes $\tau'(t) > 0$.

The preferences, which are assumed to be identical for all individuals, can be represented by a von-Neuman-Morgenstern utility function $v(C)$

$$v(C) = pu(C_E) + (1 - p)u(C_N), \quad (4)$$

where $u'(C) > 0$ and $u''(C) \leq 0$. Strict inequality in the latter implies risk aversion, which is assumed throughout.

In the basic model everyone is identical and the preferred tax rate for one person is the preferred tax rate for all. The preferred tax rate is the one that maximizes the expected utility.

The preferred tax rate is given by the first-order condition:

$$-pu'(C_E)w + (1 - p)u'(C_N)\frac{p}{1 - p}w\tau'(t^*) = 0,$$

which is equivalent with

$$MRS_{N,E} \equiv \frac{1 - p}{p} \frac{u'(C_N)}{u'(C_E)} = \frac{1 - p}{p} \frac{1}{\tau'(t^*)} \equiv P_s(t^*) \quad (5)$$

The left-hand side of (5) is the marginal rate of substitution between consumption when employed and consumption when not employed. The right-hand side is the slope of the transformation frontier between C_E and C_N , or the relative price on consumption when unemployed in terms of consumption when employed. The price is increasing in t , when $\tau''(t) < 0$. This means that the transfer costs are higher the higher the transfer. The marginal relative value of consumption when not employed in terms of the marginal value

of consumption when employed should equal the marginal relative price. The preferred degree of insurance, C_N/C_E is a decreasing function of the price. The consumer wants to transfer less consumption, the more costly this transfer is.

When there are no deadweight costs of taxation, $\tau'(t) = 1$ and $P_s = \frac{(1-p)}{p}$. The price on social insurance is constant. The relative price on consumption when not employed is equal to the relative weight on utility of consumption when not employed. If this is the case, the optimal tax is the one that gives

$$\begin{aligned} \frac{u'(C_N)}{u'(C_E)} &= 1 \Leftrightarrow \\ u'(C_E) &= u'(C_N) \end{aligned} \tag{6}$$

When we have risk aversion (6) is equivalent to

$$\begin{aligned} C_E &= C_N \Leftrightarrow (1-t)w = \frac{p}{1-p}tw \Leftrightarrow \\ t &= 1-p \Leftrightarrow C_E = C_N = pw \end{aligned}$$

With risk aversion and no deadweight cost of taxation, the optimal tax rate is the one that removes all uncertainty. The cost of risk has to be weighed against the cost of taxation. Risk aversion means that the costs of risk are positive and that the individuals prefer income smoothing. By raising the tax rate, we reduce the amount of risk, and thereby the costs of risk. However, an increased tax rate may also increase the costs of taxation. When $\tau(t) = t$, the costs of taxation are constant ($=0$). There is no trade-off, and the optimal solution will involve no risk. The social insurance system offers insurance at an actuarially fair premium under which any risk averse individual prefers complete insurance with equal consumption regardless of income-loss or not. The system allows the individuals to transfer income from income-earning state to a non-income state at a relative price equal to the relative weights on these two states.

The optimal degree of insurance, and thereby the preferred tax rate, is an increasing function of the degree of risk aversion and a decreasing function of the deadweight costs of taxation. A higher degree of risk aversion implies that for a given tax rate, the marginal relative benefit of more consumption when not employed (the left-hand side of (12)) increases. Higher deadweight costs of taxation implies that the marginal relative price on

consumption when not employed (the right-hand side of (12)) increases. In the presence of deadweight costs ($\tau'(t) < 1$) the optimal solution implies $C_E > C_N$ with $t < 1 - p$. The benefits of the insurance equal the costs at a lower degree of insurance, because the costs will be a rising function of the tax rate. The optimal solution involves risk to the individuals.

Even without deadweight costs the optimal solution may involve risk, even though the individuals are risk averse. There is a potential moral hazard problem, not modeled here. Faced with a complete insurance a person may not have any incentive to make an effort to try to keep his job. This may induce slacking and efficiency-loss and eventually lead to an increase in p . An increase in p will be a welfare loss to the population. To avoid this, the individuals have to be given work-incentives and thus be exposed to an element of risk. A condition for an optimal solution may be that $C_E > C_N$, even without a deadweight loss.

2.1 Introduction of a private insurance alternative

In what way is the solution affected by allowing for a private additional insurance? Private insurance can either be a supplement or an alternative to social insurance. I consider a "Topping up" and not a "Opting out" regime, i.e. that choosing the private insurance does not exclude the benefit of social insurance. If the tax rate is constant, the amount of social insurance is given and private insurance is an optional supplement to social insurance. However the introduction may very well affect the individual's preferences about the tax rate and may affect the amount of social insurance through a political process. If the amount of social insurance is affected, private insurance is an alternative to social insurance.

The insurance company offers insurance-cover in an amount Q at a price q . Expected profit for the insurance company is given by

$$E[\Pi] = qQ - (1 - p)Q - K, \quad (7)$$

where K is the fixed costs.

If we assume that the insurance company is risk neutral, it will choose the price that maximizes (7) given the consumers optimal choice of Q as a function of the price. We

make the assumption of risk neutrality because an insurance company has many clients and is able to pool the risks involved. This is a normal assumption in the insurance literature.

The demander has an after-tax income $I_E = (1 - t)w$ if he is employed and $I_N = \frac{p}{1-p}\tau(t)w$ if he is unemployed. He may buy additional insurance in the private market. Consumption is given by

$$C_E = I_E - qQ = (1 - t)w - qQ \quad (8)$$

$$C_N = I_N - qQ + Q = \frac{p}{1-p}\tau(t)w + Q(1 - q) \quad (9)$$

The consumer maximizes his expected utility given the tax rate, (8) and (9). The first-order condition is

$$MRS_{N,E} \equiv \frac{1-p}{p} \frac{u'(C_N)}{u'(C_E)} = \frac{q}{1-q} \equiv P_p, \quad (10)$$

if he demands additional insurance. P_p is the price in the private market on consumption when not employed in terms of consumption when employed. (10) defines the demanded amount of additional insurance as a function of the price, $Q(q)$.

The consumer demands additional insurance only if his marginal benefit of additional insurance, exceeds the costs:

$$MRS_{N,E}^0 \equiv \frac{1-p}{p} \frac{u'(I_N)}{u'(I_E)} > P_p \quad (11)$$

This is a sufficient and necessary condition for the demand to be positive, $Q(q) > 0$. If (11) does not hold, there will be no demand, and no market for additional insurance. Since the preferred degree of insurance was increasing in the degree of risk aversion and decreasing in the deadweight costs of taxation, a high degree of risk aversion and low deadweight costs of taxation makes (11) more likely to hold.

For the insurance company to be economically viable, there has to be a q that makes the demand for additional insurance positive and the expected profit non-negative. One special case is that there are no fixed costs. If (7) is to be non-negative, we have

$$qQ \geq (1-p)Q \Leftrightarrow q \geq 1-p$$

This is a necessary condition for a non-negative profit. If there are no fixed costs, it is also a sufficient condition. Any $q < 1 - p$ gives a negative expected profit, and makes the insurance company economically non-viable.

A positive demand of private insurance may have consequences for the preferred tax rate. If the costs of insurance are lower through the private insurance system than through the social insurance system, it is beneficial to make private insurance a higher share of the overall degree of insurance. Social insurance is preferred over private insurance only as long the relative price is lower. The optimal composition of insurance is the one that minimizes the costs of insurance. Social insurance is preferred up to the point where private insurance offers insurance at a lower cost. This optimal level of social insurance is implicitly given by the cost-efficient tax rate, \tilde{t} . The cost-efficient tax rate is the tax rate that equals the relative price on social and private insurance and is defined by

$$\begin{aligned} P_s(\tilde{t}) &= P_p \\ \frac{1-p}{p} \frac{1}{\tau'(\tilde{t})} &= \frac{q}{1-q} \\ \tau'(\tilde{t}) &= \frac{1-q}{q} \frac{1-p}{p} \end{aligned} \tag{12}$$

When the deadweight costs of taxation are increasing, i.e. $\tau'' < 0$, $P_s(t^*)$ is an increasing function of the tax rate. Social insurance is preferred as long as this insurance form is less costly than private insurance. By raising the tax rate, the degree of insurance rises but so does the price. If you want the degree of insurance to be higher, but this makes the price on social insurance higher than the price on private insurance, it is better to choose private than social insurance. Recall that $q \geq 1 - p$ and that we assume $\tau'(0) = 1$. Thus the right hand side of (12) is greater or equal to one. It is equal to one if $q = 1 - p$. If this is the case, $\tilde{t} = 0$. The social insurance is more expensive than the private for all values of t . For all $q > 1 - p$, there exists a level of insurance that makes private insurance more expensive than social insurance. Hence $\tilde{t} > 0$.

If the deadweight costs of taxation are constant, both the relative price on social and private insurance are constant. Private insurance is either overall preferred over social

insurance or not preferred at all. Private insurance is strictly preferred as long as

$$\tau' < \frac{1-p}{p} \frac{1-q}{q}$$

If the deadweight costs are large enough, compared to the price on private insurance, private insurance is less costly than social insurance and is strictly preferred. The cost-efficient tax rate is in this case zero. If the deadweight costs are not large enough, there exists no economically viable private insurance marked.

Consumption when not employed at the tax rate \tilde{t} is given by

$$\tilde{C}_N \equiv \frac{p}{1-p} \tau(\tilde{t}) w \quad (13)$$

\tilde{C}_N is the highest amount of consumption when not employed where social insurance is preferred over private insurance. This consumption is increasing in \tilde{t} . Any C_N higher than \tilde{C}_N is less costly in the private marked than through the social insurance system for group i . If $\tilde{t} = 0 \Leftrightarrow \tilde{C}_N = 0$. The optimal budget constraint is thus given by

$$C_E = w - P_s(t) C_N \quad \text{for } 0 \leq C_N \leq \tilde{C}_N \quad (14)$$

$$C_E = w - P_p C_N \quad \text{for } C_N > \tilde{C}_N \quad (15)$$

The optimal degree of insurance is given by

$$MRS_{N,E} = P_p \quad (16)$$

, if at optimum, there is a demand for private insurance, $Q^* > 0$, and $C_N^* > \tilde{C}_N$. If this is the case the preferred tax rate is that which equals the costs, $t^* = \tilde{t}$. If there is no demand for private insurance at optimum, $Q^* = 0$ and $C_N^* \leq \tilde{C}_N$, the optimal degree of insurance is given by

$$MRS_{N,E} = P_s(t^*) \leq P_p \quad (17)$$

Proposition 1 *If a private insurance alternative is introduced in an economy with a homogeneous population, and this private insurance is economically viable, the result is a*

reduction in the preferred tax rate and the preferred degree of social insurance

If the realized tax rate is the preferred tax rate without private insurance, then the marginal rate of substitution between the two income states equals the relative price on social insurance

$$MRS_{N,E}^0 = \frac{1-p}{p} \frac{u'(I_N)}{u'(I_E)} = P_s(t^*), \quad (18)$$

where $P_s(t)$ is defined in (5). The private insurance alternative has to offer insurance at a lower marginal cost than the social insurance does. $P_p < P_s(t^*)$ is then a necessary and sufficient condition for $Q(q) > 0$.

$$P_p = \frac{q}{1-q} < \frac{1-p}{p} \frac{1}{\tau'(t^*)} = P_s(t^*) \quad (19)$$

This is equivalent to

$$\tau'(t^*) < \frac{1-p}{p} \frac{1-q}{q}, \quad (20)$$

With no deadweight costs of taxation $\tau'(t) = 1$. Recall that for the insurance company to be economically viable, demand has to make the profits non-negative, i.e. $q \geq 1-p$ is a necessary condition for a positive supply of private insurance. Hence the right-hand side of (20) cannot be less than one. This means that deadweight costs of taxation is a necessary condition for a positive demand of private insurance and that these deadweight costs has to make social insurance more costly than private insurance. The introduction of an economically viable private insurance to an economy that has constant deadweight costs of taxation, means that the preferred tax rate is reduced to zero.

In the case of increasing deadweight costs of taxation, the necessary and sufficient condition for a viable insurance marked, eq (20), implies the condition that the tax rate has to be higher than the cost-efficient tax rate, $t^* > \tilde{t}$. This means that it is possible to buy insurance at a lower cost if the tax rate is lower. If the private insurance company offers insurance at a lower marginal cost than the social insurance does, it is beneficial to lower the amount of social insurance and acquire additional private insurance. If the marginal costs of private insurance are higher than of the social insurance, the private insurance are not economically viable.

We see from this that if the private insurance is attractive to the population, it is because private insurance offers insurance at a lower cost than social insurance. A welfare gain is then possible if we reduce the level of social insurance by reducing the tax rate. In the case of constant deadweight costs of taxation the optimal level of social insurance is zero. In the case of increasing deadweight costs, the optimal level is the one that equals the costs of the two insurance forms, defined by the tax rate \tilde{t} .

Private insurance is viable only as long as it can offer insurance at a lower cost than the social insurance system. An introduction of a private insurance alternative thus lowers the preferred tax rate in the case of a homogenous population. By introducing a private insurance alternative, we are able to minimize the costs associated with insurance. Social insurance is used for levels that makes the costs of insurance lower than through the private insurance.

3 Heterogeneous risk

I now expand the model to a model where the population is divided into three groups, H, M and L. This introduces aspects of conflict of interests. The groups are different with respect to the risk of income-loss. The share σ_i of the population has a probability p_i of keeping their job, with $0 < p_i < 1$, $i = H, M, L$ and $p_H > p_M > p_L$. I assume $\sigma_H < \frac{1}{2}$ and $\sigma_L < \frac{1}{2}$, which means that the median risk-holder is in the middle group.

Total tax-income is now

$$T(t) = (p_H\sigma_H + p_M\sigma_M + p_L\sigma_L) \tau(t)w = \bar{p}\tau(t)w \quad (21)$$

\bar{p} is the average probability of a person keeping the job, which I assume is identical to the probability in the basic model.

A member of group i has an utility-function:

$$v_i(C) = p_i u(C_E) + (1 - p_i) u(C_N), \quad (22)$$

with the same properties as above.

This member has a preferred tax rate that maximizes his utility. The first-order

condition is:

$$-p_i u'(C_E)w + (1 - p_i)u'(C_N)\frac{\bar{p}}{1 - \bar{p}}w\tau'(t) = 0$$

$$MRS_{N,E}^i \equiv \frac{1 - p_i}{p_i} \frac{u'(C_N)}{u'(C_E)} = \frac{1 - \bar{p}}{\bar{p}} \frac{1}{\tau'(t_i^*)} \equiv P_s \quad (23)$$

The price is identical to the previous section, but the preferences vary. More specifically, the weights put on utility of consumption whether one is employed or not employed, vary. The weight put on utility of consumption when not employed is greater, the greater the probability of losing the job. We see from the first-order condition that if $p_i = \bar{p}$, i.e. the probability of keeping the job for group i equals the average probability, the preferred tax rate is identical to the solution with homogenous risk, e.g. (5). The lowest probability has to be lower than the average probability, $p_L < \bar{p}$, and the highest probability has to be greater than average, $p_H > \bar{p}$. Hence, the L -group faces a relative price higher, and the H -group a relative price lower, than their respective relative weight on consumption. The preferred tax rate for the H -group is thus lower than in the previous section with homogenous risk and the preferred tax rate for L -group is higher. The median preferred tax rate is t_M^* . The median voter is thus in the M -group and the realized tax rate is t_M^* . If $p_M = \bar{p}$ the realized tax rate will be identical to the previous section. Without deadweight costs of taxation ($\tau'(t) = 1$), the preferred tax rate is the one that gives $C_E = C_N$, which is $t = 1 - \bar{p}$. Since $p_L < \bar{p}$, members of the L -group will prefer a higher tax rate and over-insurance. A member of the H -group prefers a lower tax rate, since $p_H > \bar{p}$, and thus incomplete insurance. The reason for this is that in a social insurance system the ones facing a low risk are subsidizing the ones with a high risk. The presence of deadweight costs will lower the preferred tax rate for all three groups. When $\tau'' < 0$, the reduction is largest for the highest preferred tax rate, which is the tax rate preferred by the low-risk group, since this tax rate has the highest costs. The cost of transferring income from one state to the other is the same as above, but the weight on the two states is given in the preferences are heterogeneous. Hence the benefits equal the costs at different levels of insurance.

3.1 Introduction of a private alternative

We see that when $\sigma_H < 1/2$ and $\sigma_L < 1/2$, the realized tax rate is t_M^* , which is lower than the preferred tax rate for the L-group and higher than the preferred tax rate for the H-group. The realized degree of social insurance is thus lower than preferred for the L-group, and this may lead to a demand for an optional additional private insurance. However the L-group is the group with the highest probability of losing their job. The members of this group are thereby the least attractive customers for the private insurance company. If the insurance company can observe the risk of each group, the price offered for private insurance is higher the higher the risk of losing the job. If the risks cannot be observed, we have an asymmetric information situation. If this is the case, the appropriate solution is a perfect Bayesian equilibrium. My focus is not on informational aspects though, and I limit the subject by assuming that the risks are observable.

In what way is the solution affected by allowing for a private additional insurance? Private insurance can either be a supplement or an alternative to social insurance. As before, I assume that it is offered as a supplement. The insurance company offers insurance-cover to group i in an amount Q_i at a price q_i . Expected profit for the insurance company for group i is given by

$$E[\Pi_i] = p_i q_i Q_i + (1 - p_i)(q_i Q_i - Q_i) - K = q_i Q_i - (1 - p_i)Q_i - K, \quad (24)$$

where K is the fixed costs. We assume that the insurance company is risk neutral, and chooses the price that maximizes (24) given the consumers optimal choice of Q_i as a function of the price. As in section two, a necessary condition for non-negative profits for each group of customers is that $q_i \geq 1 - p_i$.

The demander has an after-tax income $I_E = (1 - t)w$ if he is employed and $I_N = \frac{\bar{p}}{1 - \bar{p}}\tau(t)w$ if he is unemployed. He may buy additional insurance in the private market. Consumption is then given by

$$C_E^i = I_E - q_i Q_i = (1 - t)w - q_i Q_i \quad (25)$$

$$C_N^i = I_N - q_i Q + Q_i = \frac{\bar{p}}{1 - \bar{p}}\tau(t)w + Q_i(1 - q_i) \quad (26)$$

The consumer maximizes his expected utility given the tax rate, (25) and (26). The

first-order condition is

$$MRS_{N,E}^i = \frac{1 - p_i}{p_i} \frac{u'(C_N)}{u'(C_E)} = \frac{q_i}{1 - q_i} \equiv P_p^i, \quad (27)$$

if he demands additional insurance. P_p is the price in the private market on consumption when not employed in terms of consumption when employed. (27) defines the demanded amount of additional insurance as a function of the price, $Q_i(q_i)$.

The price on private insurance varies from group to group, the price being lower, the lower the probability of keeping their job, p_i

$$q_L > q_M > q_H,$$

which in turn makes the marginal relative price on more consumption when not employed lower, the lower the probability

$$P_p^L > P_p^M > P_p^H$$

Group i has a positive demand as long as their marginal value of additional insurance at the realized tax rate, t^* , exceeds the marginal relative price

$$MRS_{N,E}^i(t^*) > P_p^i \Leftrightarrow Q_i(q_i) > 0$$

If this is the case, the optimal amount of private insurance is given from (27).

A positive demand of private insurance may have consequences for the preferred tax rate. If the costs of insurance are lower through the private insurance system than through the social insurance system, it is beneficial to make private insurance a higher share of the overall degree of insurance. Social insurance is preferred over private insurance only as long the relative price is lower. The optimal composition of the insurance is the one that minimizes the costs of insurance. The social insurance is preferred up to the point where the private insurance offers insurance at a lower cost. If optimal demand for private insurance is positive, the optimal level of social insurance is implicitly given by the cost-efficient tax rate, \tilde{t}_i . The cost-efficient tax rate is the tax rate that equals the relative

price on social and private insurance and is defined by

$$\begin{aligned}
P_s(\tilde{t}_i) &= P_p^i \\
\frac{\bar{p}}{1 - \bar{p}} \frac{1}{\tau'(\tilde{t}_i)} &= \frac{q_i}{1 - q_i} \\
\tau'(\tilde{t}_i) &= \frac{1 - q_i}{q_i} \frac{1 - \bar{p}}{\bar{p}}
\end{aligned} \tag{28}$$

When the deadweight costs of taxation are increasing, i.e. $\tau'' < 0$, $P_s(t^*)$ is an increasing function of the tax rate. Social insurance is preferred as long as this insurance form is less costly than private insurance. By raising the tax rate, the degree of insurance rises but so does the price. If you want the degree of insurance to be higher, but this makes the price on social insurance higher than the price on private insurance, it is better to choose private than social insurance. The lower the price on private insurance is, the lower is the cost-efficient tax rate. Since the group with the lowest probability of losing their job is offered the lowest price we have

$$\tilde{t}_L > \tilde{t}_M > \tilde{t}_H$$

We remember that $q_i \geq 1 - p_i$, $p_H > p_M = \bar{p} > p_L$, and $\tau'(0) = 1$. Thus for $i = L$, the right-hand side of (28) is greater than one. For $i = M$, it is equal to one if $q_M = 1 - \bar{p}$. Hence $\tilde{t}_L > 0$, $\tilde{t}_M = 0$ iff $q_M = 1 - \bar{p}$ and $\tilde{t}_H = 0$ for some $q_H > 1 - p_H$. An actuarially fair premium on private insurance is not sufficient for the L-group to want remove the social insurance altogether, it is necessary and sufficient for the M-group and it sufficient, but not necessary for the H-group. The reason for this is that the H-group in a way is subsidizing the L-group through the social insurance system and that there are no such effects in the private insurance system.

If the preferred tax rate without an additional private insurance is lower than the cost-efficient tax rate, the optimal demand for private insurance is non-positive. The preferred tax rate is then unaffected by the private alternative. If it is higher, the introduction of a private alternative lowers the preferred tax rate to its cost-efficient level.

What the effects of the introduction of a private alternative are, depends on whether the different groups have a positive optimal demand for private insurance or not. Although the L-group is insured in a lower degree in the social system than preferred, and may thus

demand additional private insurance, it does not necessarily mean that their optimal demand is positive. For this group, the optimal solution could just be a higher degree of social insurance, but this is not realized because the degree of social insurance is collectively decided. If there are no deadweight costs of taxation, there exists no private insurance alternative that can compete with the social insurance and make a non-negative profit.

The H-group prefers a lower level of social insurance than realized and has to be offered a price on private insurance that is sufficiently lower than the marginal relative costs of social insurance without the private alternative. Hence this group may not demand additional insurance even if it is attractive to the L-group which faces lower marginal relative costs of social insurance. Even so, the preferred tax rate may be affected by the introduction of the private alternative. The optimal insurance composition may involve private insurance, but the optimal degree of private insurance may be zero if the degree of social insurance is higher than optimal.

For the M-group, the situation is as in section two. If there is a positive demand for private insurance for this group, it is because the price offered on private insurance is lower than the price on social insurance, at the preferred degree of social insurance without a private alternative. Hence, if this group demands additional private insurance for $t = t_M^*$, their preferred tax rate is lowered to \tilde{t}_M .

In the following I assume that the private insurance marked is such that the price offered makes optimal demand for private insurance positive for the H- and M-groups, but not for the L-group. This may be the case if the impact of the subsidizing effects in the social insurance are large (p_H/p_L is large) relative to the costs of taxation for the L-group, and thus involves no private insurance in the optimal solution.

A positive optimal demand for private insurance for the M- and H-group implies that the preferred tax rate is the cost-efficient tax rate

$$t_M^* = \tilde{t}_M > \tilde{t}_H = t_H^*$$

The preferred tax rate for the L-group is as before given by the trade-off between the costs and benefits of social insurance. We remember that this tax rate was higher than the preferred tax rate for the two other groups without a private alternative, because of

the subsidizing effects and that if the optimal demand for private insurance is positive, this reduces the preferred tax rate to the cost-efficient level.

If both the M- and H-group have a positive optimal demand for private insurance, the introduction of the private alternative implies a majority in favor of reducing the tax rate, as long as $\sigma_L < 1/2$. This implies a welfare loss for the L-group. Lowering the tax rate means that the benefits of the subsidizing effects are reduced and that the preferred degree of insurance has to involve a higher degree of private insurance, which is more costly for this group than social insurance.

This shows that even if it is in the interests of the L-group to acquire an additional private insurance, because the degree of social insurance is lower than optimal, an introduction of a private insurance alternative may harm this group.

4 Heterogeneous income

In this version of the model the three groups have the same probability of keeping their jobs, p , but different wages, $w_H > w_M > w_L$. Total tax-income is now

$$T(t) = \tau(t) (\sigma_H w_H + \sigma_M w_M + \sigma_L w_L) p = \tau(t) \bar{w} p, \quad (29)$$

where

$$\bar{w} = \sigma_H w_H + \sigma_M w_M + \sigma_L w_L \quad (30)$$

is the average wage for those employed. For comparison, I assume that the average wage is equal to the median wage, $\bar{w} = w_M$, and that this is equal to the wage in section two.

The consumption of an employed person in group i is given by.

$$C_E^i = (1 - t) w_i \quad (31)$$

Consumption of an unemployed person is as before and unrelated to prior earnings. Preferences of a member of group i is represented by the utility-function

$$v_i(C) = p u(C_E^i) + (1 - p) u(C_N) \quad (32)$$

The first-order condition for the optimal tax rate for a member of group i :

$$-pu'(C_E^i)w_i + (1-p)u'(C_N)\frac{p}{1-p}\tau'(t)\bar{w} = 0$$

$$MRS_{N,i} \equiv \frac{1-p}{p} \frac{u'(C_N)}{u'(C_E^i)} = \frac{1-p}{p} \frac{w_i}{\bar{w}} \frac{1}{\tau'(t)} \equiv P_s^i(t_i^*) \quad (33)$$

The price on consumption when not employed in terms of consumption when employed is no longer identical for all individuals, but increasing in w_i . That means that for a given t , $P_s^H > P_s^M > P_s^L$. High-wage earners has to pay more for one unit of consumption when not employed in terms of consumption when employed and thus prefer a lower degree of insurance, C_N/C_E . This does not necessarily mean that the preferred tax rate is decreasing in the wage. High-wage earners have a higher initial level of consumption when employed and may have a higher preferred level of consumption when not employed although the preferred degree of insurance is lower. As in section two, the preferred degree of insurance, and thereby t_i^* , is increasing in the degree of risk aversion and decreasing in the deadweight costs of taxation.

What are the effects of a mean preserving spread in the wages, i.e. that w_H increases, w_L decreases with \bar{w} is constant? We see that this increases P_c^H and decreases P_c^L . A mean preserving spread thus reduces the preferred degree of insurance and tax rate for the high income earners and increases the preferred degree of insurance and tax rate for the low income earners.

We view the preferred tax rates in Figure 1 and 2, showing the marginal costs and benefits of the welfare-state as functions of the tax rate.

The benefits are the same for all three groups, since unemployment-benefit is unrelated to prior earnings. The marginal benefits of the tax rate is measured by $pu'(C_N)\tau'(t)\bar{w}$, with

$$\frac{\partial pu'(C_N)\tau'(t)w_M}{\partial t} = pu''(C_N)\tau'(t)\bar{w}^2 + pu'(C_N)\tau''(t)\bar{w} < 0 \quad (34)$$

The benefits are a decreasing function of the tax rate if $\tau'' \leq 0$ and we have risk aversion, $u''(C) < 0$. The benefit from the last amount taxed, in terms of value of consumption

when not employed, is lower the higher the tax rate is. This is the same for Figure 1 and 2.

The marginal costs are measured by $pu'(C_E^i)w_i$, with

$$\frac{\partial pu'(C_E^i)w_i}{\partial t} = -pu''(C_E^i)w_i^2 > 0 \quad (35)$$

The costs are increasing in the tax rate, as long as $u'' < 0$.

What is the relation between the wage and the costs?

$$\frac{\partial pu'(C_E^i)w_i}{\partial w_i} = p \frac{u''(C_E^i)(1-t)w_i + u'(C_E^i)}{u'(C_E^i)} \quad (36)$$

The coefficient of relative risk aversion, μ , (which is assumed constant here) is defined as

$$\mu \equiv -\frac{u''(C)}{u'(C)}C$$

According to Arrow (1965) the relative risk aversion is the elasticity of the marginal utility of wealth.

If we insert for $u''(C) = -\frac{\mu}{C}u'(C)$ and $C_E^i = (1-t)w_i$ in (36), we get

$$\frac{\partial pu'(C_E^i)w_i}{\partial w_i} = pu'(C_E^i)[1 - \mu] \quad (37)$$

The marginal costs of the tax rate are lower, the higher the wage if $\mu > 1$. Moreover

$$\frac{\partial^2 pu'(C_E^i)w_i}{\partial w_i \partial t} = -pu''(C_E^i)w_i[1 - \mu] < 0 \quad (38)$$

,when $\mu > 1$ and $u''(C_E^i) < 0$.

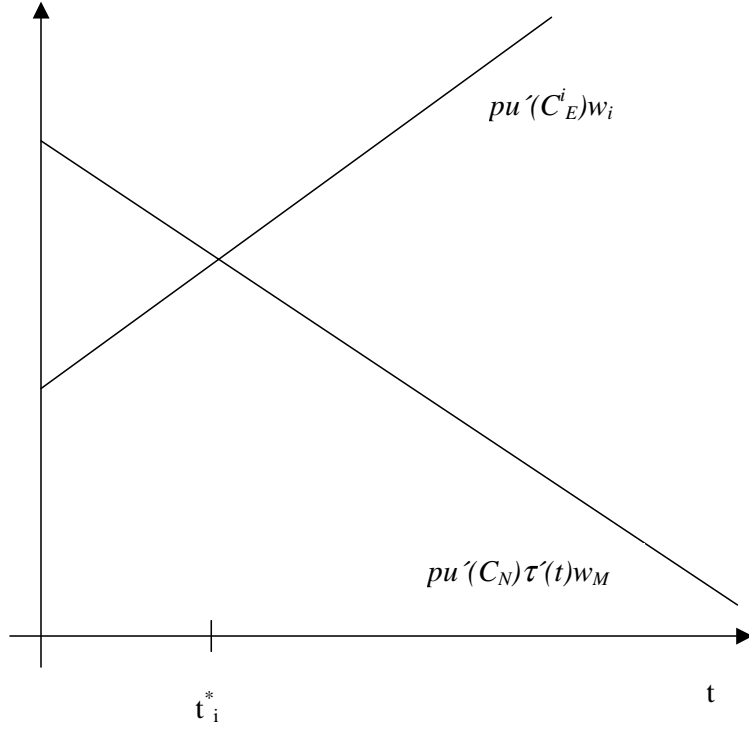


Figure 1: $\mu = 1, i = L, M, H$

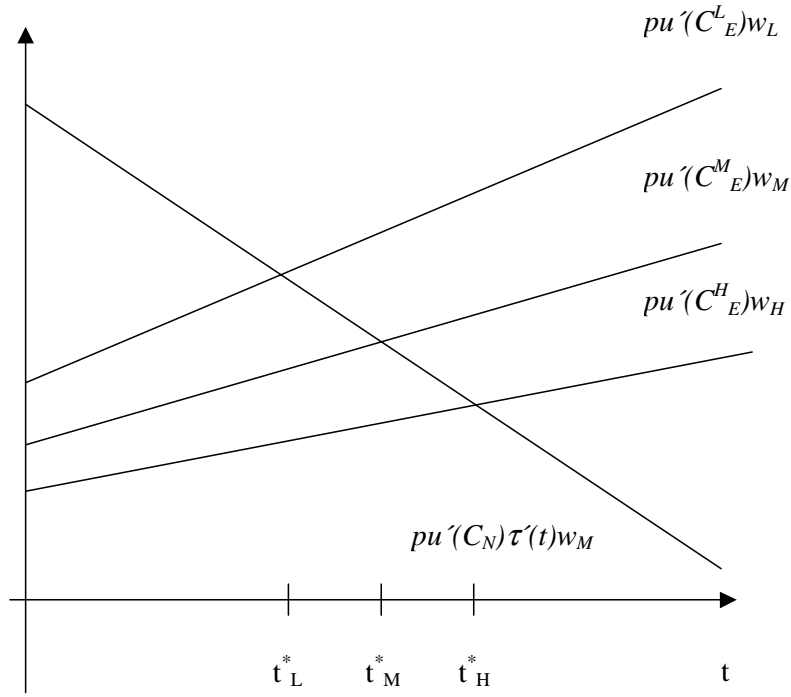


Figure 2: $\mu > 1$

Figure 1 shows the case when $\mu = 1$ as in the case when $u(C) = \ln C$. In this case $pu'(C_E^i)w_i = \frac{p}{(1-t)}$ and the three cost-curves are identical. The marginal costs of each dollar taxed are unrelated to the pre-tax income. If the benefits and the costs of every dollar taxed are the same for all three groups, the preferred tax rate is the same and we have no conflicts of interests. This means that $t_i^* = t^*$ and

$$MRS_{N,i} = P_S^i(t^*) \quad (39)$$

for all three groups.

Figure 2 shows the case when $\mu > 1$. The different cost-curves are lower, the higher the income and the differences are increasing. An income increase has two effects on the marginal costs of the tax rate: one positive and one negative. The positive effect is that higher income leads to higher consumption when employed, making the tax more costly. The negative one is that more income reduces the marginal utility of consumption when employed when we have risk aversion, making the tax less costly. When $\mu > 1$, as seen in Figure 2, the negative effect dominates, making the marginal costs of the tax rate decreasing in the wage, with increasing differences. Every dollar taxed is more costly to the low income earners and their costs are increasing more rapidly than for higher income levels:

$$pu'(C_E^H)w_H < pu'(C_E^M)w_M < pu'(C_E^L)w_L \quad (40)$$

$$\frac{\partial pu'(C_E^L)w_L}{\partial t} > \frac{\partial pu'(C_E^M)w_M}{\partial t} > \frac{\partial pu'(C_E^H)w_H}{\partial t} \quad (41)$$

Thus, if $\mu > 1$, the marginal benefits of the tax rate equals the marginal costs at a higher level the higher income is. The preferred tax rate is an increasing function of the wage

$$t_H^* > t_M^* > t_L^*$$

This means that the demand for social insurance is increasing in income even though the cost of social insurance is increasing in income as well. This means that insurance is a normal good, with demand increasing when income increases.

Moreover $w_H/w_M > 1$ and $w_L/w_M < 1$. This implies, if $\tau''(t) \leq 0$, that

$$P_s^H(t_H^*) > P_s^M(t_M^*) > P_c^L(t_L^*)$$

If $\mu > 1$, we thus have conflicts of interests, with $t_H^* > t_M^* > t_L^*$. If $\sigma_L < 1/2$ and $\sigma_H < 1/2$, the median voter is in the M-group and the realized tax rate is t_M^* . The realized tax rate is thus lower than the preferred tax rate for the high-wage group, and higher than preferred for the low-wage group, even though the costs of insurance are higher for the high income group. The high-wage earners prefers a higher degree of insurance, i.e. a higher tax rate and the low-wage earners prefer a lower degree of insurance, i.e. a lower tax rate:

$$MRS_{N,L} < P_c^L(t^*)$$

$$MRS_{N,M} = P_s^M(t^*)$$

$$MRS_{N,H} > P_s^H(t^*)$$

We see that when

$$w_M = \bar{w} \Leftrightarrow P_s^M = \frac{p}{1-p} \frac{1}{\tau'(t)} = P_s,$$

P_s being the price in the section with a homogeneous population. Then (33) for $i = M$ is identical to (5) and the realized tax rate is identical to the one in section one.

With no deadweight costs of taxation, $\tau(t) = t$,

$$P_s^i = \frac{1-p}{p} \frac{w_i}{\bar{w}}$$

The price is the relative weight on consumption multiplied by the wage relative to the average wage. If $w_i = w_M$, the price is equal to the relative weight on consumption and a risk averse individual prefers perfect insurance. Remember that $w_H > w_M > w_L$. Therefore, the L -group prefers over-insurance and the H -group prefers incomplete insurance, when there are no deadweight costs of taxation. The reason for this is the redistributive element of the social insurance. The high-wage earners are subsidizing the

low-wage earners in the social insurance system. With no deadweight costs of taxation we have the following for the M -group:

$$\bar{w} = w_M \Leftrightarrow P_s^M = \frac{1-p}{p} \Leftrightarrow C_N = C_E^M$$

With $\bar{w} = w_M$ and no deadweight cost, $P_s^M = \frac{1-p}{p}$, the social insurance system offers insurance at an actuarially fair premium for the median voter. He is able to transfer consumption from when being employed to when not being employed at a relative price equal to the relative weight on consumption. If the individual is risk averse, he prefers to have equal consumption regardless of his employment status. The individuals prefer a lower degree of insurance the higher the price.

4.1 Introduction of a private insurance alternative

In what way is the preferred tax rates and hence possibly the realized tax rate, affected by the introduction of a private alternative? As in section two I assume that the insurance company maximizes expected profit. I assume that there is no possibility of price discrimination. Insurance is offered at a price independent of income.

The demander has an after-tax income $I_E^i = (1-t)w_i$ if he is employed and $I_N = \frac{p}{1-p}\tau(t)\bar{w}$ if he is unemployed. He may buy additional insurance in the private market. Consumption is given by

$$C_E^i = I_E^i - qQ_i = (1-t)w_i - qQ_i \quad (42)$$

$$C_N^i = I_N - qQ_i + Q_i = \frac{p}{1-p}\tau(t)\bar{w} + Q_i(1-q) \quad (43)$$

Section two showed that the consumers choose a level of insurance that makes the marginal rate of substitution between the two income states equal to the relative price on private insurance:

$$MRS_{N,i} = \frac{1-p}{p} \frac{u'(C_N)}{u'(C_E^i)} = \frac{q}{1-q} \equiv P_p \quad (44)$$

if he demands additional insurance. P_p is the same price as in section 2. (44) defines the demanded amount of additional insurance for group i as a function of the price, $Q_i(q)$.

As before, a necessary condition for a non-negative profits that $q \geq 1 - p$. If there are no fixed costs, it is also a sufficient condition. Any $q < 1 - p$ gives a negative expected profit, and makes the insurance company economically non-viable.

Proposition 2 *Private insurance is more attractive the higher the income.*

The consumer demands additional insurance only if his marginal value of consumption when not employed in terms of his marginal value of consumption when employed at the initial level, exceeds the price.

$$MRS_{N,i}^0 \equiv \frac{1-p}{p} \frac{u'(I_N)}{u'(I_E^i)} > P_p \quad (45)$$

This is a sufficient and necessary condition for the demand for group i to be positive, $Q_i > 0$. If (45) does not hold for $i = H, M, L$, there will be no demand, and no marked for additional insurance. Because the initial income is higher, the higher the wage, we have that

$$MRS_{N,H}^0 > MRS_{N,M}^0 > MRS_{N,L}^0$$

Hence, if the high income earners have a non-positive demand for private insurance, the demand is non-positive for the two other groups as well. Moreover, if the demand is positive for the low income earners, it is positive for the two other groups as well.

If $\mu = 1$, the realized tax rate, t^* , is optimal for all three groups. If this is the realized tax rate as well, we have:

$$MRS_{N,i}^0 = \frac{1-p}{p} \frac{u'(I_N)}{u'(I_E^i)} = P_s^i(t^*) \quad (46)$$

where $P_s^i(t)$ is defined in (33). $P_p < P_s^i(t^*)$ is then a necessary and sufficient condition for $Q_i(q) > 0$.

The H-group pays the highest price on social insurance, $P_s^H(t^*) > P_s^M(t^*) > P_s^L(t^*)$. Hence for $\mu = 1$, a necessary and sufficient condition for a positive demand for private insurance is that $P_p < P_s^H(t^*)$. The price on consumption when not employed through the private insurance system has to be lower than the price the H-group pays through the social insurance system. For the insurance company to be economically viable this

demand has to make the profits non-negative.

With no fixed costs in the insurance marked, the lowest q that makes the profits non-negative is $q = 1 - p$. Then positive demand requires

$$P_p = \frac{q}{1-q} = \frac{1-p}{p} < \frac{1-p}{p} \frac{w_H}{\bar{w}} \frac{1}{\tau'(t^*)} = P_s^H(t^*)$$

Recall that $\frac{w_H}{\bar{w}} > 1$ and $\tau'(t^*) \leq 1$. There are redistributive effects in the social insurance system. A worker earning a higher-than-average wage is subsidizing the workers earning a lower than average wage in the social insurance system. In the private insurance system there are no subsidizing effects. The private insurance system can thus offer a price that is lower for the high-wage earners than the price the social insurance system offers, depending on the size of the fixed costs in the private marked. A sufficient condition for an economically viable insurance marked, is that there are no fixed costs in the insurance marked, even when there are no deadweight costs of taxation.

If $\mu > 1$, the previous section showed that the preferred tax rate is increasing in the wage. If the median voter is in the M-group, the realized tax rate is t_M^* , the H-group prefers a higher tax rate and the L-group prefers a lower tax rate. Both the H- and L-group are dissatisfied with the amount of social insurance. At the realized relative price on consumption when not employed, the H-group prefers a higher, and the L-group a lower degree of insurance than offered through the social insurance system. The relative value of more consumption when not employed is higher than the costs for the H-group and lower for the L-group. In other words, the marginal rate of substitution is higher than the price for the H-group and lower than the price for the L-group:

$$MRS_{N,H}^0 > P_s^H(t_M^*) > MRS_{N,M}^0 = P_s^M(t_M^*) > P_c^L(t_M^*) > MRS_{N,L}^0$$

Then $P_p < P_s^H(t^*)$ is no longer a necessary condition for $Q_H(q) > 0$, but a sufficient one. We may have $P_p > P_s^H(t_M^*)$ and still have $MRS_{N,H}^0 > P_p$, thus making the demand for additional insurance positive. With $\mu > 1$, the necessary and sufficient condition for viability is that $MRS_{N,H}^0 > P_p$ and that this demand makes the profits non-negative. The realized tax rate is the optimal tax rate for the M-group and $P_p < P_s^M(t^*)$ is still a necessary and sufficient condition for the demand for the M-group to be positive. $P_p < P_c^L(t_M^*)$ is a necessary condition for a positive demand for the L-group, but not a sufficient

one, since this group is paying a tax higher than optimal and thus a higher degree of insurance than optimal at the given price.

As in section two, an introduction of a private insurance is likely to affect the preferred tax rate for the three groups. If the private alternative offers insurance that is less costly, we expect a reduction in the preferred tax rate. Social insurance is preferred as long as the price on consumption when not employed is lower through the social insurance system than through the private insurance system ($P_p > P_s^i(t_i^*)$). If there are no deadweight costs of taxation, the price through the social insurance system is a constant, and is either greater or smaller than the price offered through the private insurance system for all tax rates. The L-group is being subsidized in the social insurance system, paying an actuarially more-than-fair premium without any deadweight costs. No private alternative can compete with that. The M-group is paying an actuarially fair premium in the social insurance system, and the best the private insurance company can do is to offer the same price. This makes the M-group indifferent in regards to the two insurance forms, but leaves the insurance company with a non-positive profit (negative in the presence of fixed costs). With no deadweight costs of taxation, the only group who may benefit on private insurance, is the H-group, who is paying an actuarially less-than-fair premium.

When we have increasing deadweight costs of taxation, $P_s^i(t)$ is no longer a constant, but an increasing function of the tax rate. Social insurance is preferred as long as this insurance form is cheaper than private insurance. By raising the tax rate, the degree of insurance rises but so does the price. If you want the degree of insurance to be higher, but this makes the price on social insurance higher than the price on private insurance, it is better to choose private than social insurance.

Proposition 3 *The cost-efficient tax rate is higher the lower the income*

The cost-efficient tax rate for group i, \tilde{t}_i , is the one that equals the two prices and is defined by:

$$\begin{aligned} P_s^i(\tilde{t}_i) &= P_p \\ \frac{1-p}{p} \frac{w_i}{\bar{w}} \frac{1}{\tau'(\tilde{t}_i)} &= \frac{q}{1-q} \\ \tau'(\tilde{t}_i) &= \frac{1-q}{q} \frac{1-p}{p} \frac{w_i}{\bar{w}} \end{aligned} \tag{47}$$

$\tau'' \leq 0$ and $w_H > w_M = \bar{w} > w_L$. This means that

$$\tilde{t}_L > \tilde{t}_M > \tilde{t}_H$$

If the right hand side of (47) is greater or equal to one, social insurance is more expensive than private insurance for all values of t , since $\tau'(0) = 1$. If this is the case, $\tilde{t}_i = 0$. We remember that $q \geq 1 - p$. This means that $\tilde{t}_L > 0$ and $\tilde{t}_M > 0$ if $q > 1 - p$. A sufficient and necessary condition for $\tilde{t}_H = 0$ is

$$\begin{aligned} \frac{1-q}{q} \frac{1-p}{p} \frac{w_H}{\bar{w}} &\geq 1 \\ q &\leq \frac{(1-p)w_H}{(1-p)w_H + p\bar{w}} > 1 - p \end{aligned} \quad (48)$$

This means that the preferred tax rate for the H-group is zero even for an actuarially less-than-fair premium on private insurance.

As before, consumption when not employed at this tax rate is given by

$$\tilde{C}_N^i \equiv \frac{p}{1-p} \tau(\tilde{t}_i) \bar{w} \quad (49)$$

\tilde{C}_N^i is the highest amount of consumption when not employed where social insurance is preferred over private insurance for group i . This consumption is increasing in \tilde{t}_i and thus decreasing in w_i , i.e. $\tilde{C}_N^L > \tilde{C}_N^M > \tilde{C}_N^H$. Any C_N higher than \tilde{C}_N^i is less costly in the private market than through the social insurance system for group i . If $\tilde{t}_i = 0 \Leftrightarrow \tilde{C}_N^i = 0$. The optimal budget constraint for group i is thus given by

$$C_E^i = w_i - P_s^i(t) C_N \quad \text{for } 0 \leq C_N \leq \tilde{C}_N^i \quad (50)$$

$$C_E^i = w_i - P_p C_N \quad \text{for } C_N > \tilde{C}_N^i \quad (51)$$

The optimal degree of insurance is given by

$$MRS_{N,i} = P_p, \quad (52)$$

if at optimum, there is a demand for private insurance, i.e. $Q_i^* > 0$, and $C_N^* > \tilde{C}_N^i$. If this is the case the preferred tax rate is that which equals the costs, $t_i^* = \tilde{t}_i$. If there is no

demand for private insurance at optimum, i.e. $Q_1^* = 0$ and $C_N^* \leq \tilde{C}_N^i$, the optimal degree of insurance is given by

$$MRS_{N,i} = P_s^i(t_i^*) \leq P_p \quad (53)$$

If this is the case, the optimal tax rate is unaffected by the introduction of the private alternative and makes the price on the social insurance lower than on the private, $t_i^* < \tilde{t}_i$.

We remember that a mean preserving spread in the wages increases P_s^H and decreases P_s^L . Increasing wage inequality thus makes private insurance more attractive to the high-income earners and less attractive to the low-income earners.

In the case with increasing deadweight costs of taxation, the costs of more consumption when not employed is increasing when insured socially. If, at $C_N = \tilde{C}_N^i$, the marginal relative value of more consumption when not employed exceeds the price, it would increase welfare to increase the degree of insurance. However, it is less costly to get the additional insurance in the private market because of the increasing deadweight costs of taxation. If the optimal degree of taxation for group i implies that $C_N > \tilde{C}_N^i$, the preferred tax rate is $t_i^* = \tilde{t}_i$. If the preferred tax rate implies that $C_N < \tilde{C}_N^i$, the preferred tax rate is given by (53). As in section two, individuals that have a positive optimal demand for private insurance have a lower preferred tax rate than without a private insurance alternative.

One important aspect of this insight is that $\mu > 1$ no longer is a necessary condition for conflicts of interests. If at least one group has a positive optimal demand for private insurance, the preferred tax rates are no longer identical for the three groups. The group(s) having a positive optimal demand, has a preferred tax rate differing from the ones without a positive optimal demand. If all three groups have a positive demand, the preferred tax rate is a decreasing function of the wage, $\tilde{t}_L > \tilde{t}_M > \tilde{t}_H$.

If the introduction of a private insurance alternative affects the preferred tax rate, the realized tax rate may be affected as well. There may be a majority in favor of lowering the tax rate even though private insurance is not attractive to a majority. The effect on the realized tax rate depends among other things on which of the three groups have positive optimal demands for private insurance. Proposition 2 stated that private insurance is more attractive the higher the income. We thus have three cases to consider: only the high-income earners have a positive optimal demand, both the high- and median-income

earners have a positive optimal demand, all three groups have a positive optimal demand.

4.1.1 Only the high-income earners have a positive optimal demand

A positive optimal demand for additional private insurance by only the high income earners, means that

$$P_s^H(t_H^* = \tilde{t}_H) = P_p > P_s^M(t_M^*) > P_c^L(t_L^*) \quad (54)$$

It follows that t_M^* and t_L^* are unaffected by the private alternative and that the preferred tax rate for the H-group, t_H^* , is lower than the preferred tax rate without the private alternative. An introduction of a private supplemental insurance leads to a gain for the high-income group from two potential sources. Firstly, private insurance makes it possible to get additional insurance at the given level of the tax rate. Secondly, a reduction in the preferred tax rate may lead to a reduction in the realized tax rate. If there is a reduction in the realized tax rate, the high-income earner is able to switch from social insurance to the, for them, less costly private insurance. If $\mu = 1$, the preferred tax rates were identical without private insurance. A reduction in the preferred tax rate for the high income earners thus makes this tax rate lower than the preferred tax rates for the two other groups:

$$t_L^* = t_M^* > t_H^*$$

The private alternative then has no effect on the realized tax rate as long as $\sigma_H < 1/2$. The only effect is that the high-income earners demands additional insurance and are made better off.

If $\mu > 1$, the preferred tax rates were increasing in the wage without private insurance. The low-income earners wish to reduce the tax rate. If $\sigma_M < 1/2$, the reduction in the preferred tax rate for the high income earners following the introduction of the private insurance alternative may affect the realized tax rate.

Proposition 4 *The introduction of a private insurance that is only attractive to the share of the population with the highest income, implies a coalition of the high- and low-income earners and a majority in favor of lowering the tax rate, as long as $\sigma_M < 1/2$ and $\mu > 1$.*

If $\sigma_M < 1/2$, the introduction of the private insurance alternative shifts the political center from the median income earners as long as $t_H^* < t_M^*$.

t_H^* is defined from

$$\frac{1-p}{p} \frac{1}{\tau'(t_H^*)} \frac{w_H}{w_M} = \frac{q}{1-q} \quad (55)$$

From this expression we get that

$$\frac{1-p}{p} = \frac{q}{1-q} \frac{w_M}{w_H} \tau'(t_H^*) \quad (56)$$

$P_s^M(t_M^*)$ is defined from

$$P_s^M(t_M^*) = \frac{1-p}{p} \frac{1}{\tau'(t_M^*)} \quad (57)$$

We insert for $\frac{1-p}{p}$ from (56) and get

$$\begin{aligned} P_s^M(t_M^*) &= \frac{q}{1-q} \frac{w_M}{w_H} \frac{\tau'(t_H^*)}{\tau'(t_M^*)} \Rightarrow \\ \frac{\tau'(t_H^*)}{\tau'(t_M^*)} &= \frac{1-q}{q} P_s^M(t_M^*) \frac{w_H}{w_M} \Rightarrow \\ \frac{\tau'(t_H^*)}{\tau'(t_M^*)} &= \frac{P_s^H(t_M^*)}{P_p} \end{aligned} \quad (58)$$

Since $\tau'' < 0$ in the case of increasing deadweight costs, $t_H^* < t_M^*$ as long as the right-hand side of (58) is greater than one. This is equivalent to

$$P_s^H(t_M^*) > P_p \quad (59)$$

Which is the necessary and sufficient condition for a positive demand for private insurance.

The equivalent condition for the introduction to lower the tax rate of the high income earners below that of the low income earners is:

$$P_s^H(t_L^*) > P_p \quad (60)$$

We remember that for $\mu > 1$, $t_M^* > t_L^*$ without private insurance. Condition (60) is

thus stricter than condition (59) for $\mu > 1$. Condition (60) is more likely to hold the greater the wage inequality. For $\mu = 1$, the two conditions are equivalent ($t_M^* = t_L^*$).

When $\mu > 1$, the introduction of a private insurance alternative leads to an Ends-against-mean situation. It is both in the high- and low-income earners interests to lower the tax rate. As long as $\sigma_M < 1/2$, this means that the median voter falls outside the M-group. This can be seen as a loss of political power for this group. The realized tax rate is thus no longer the optimal tax rate for this group. This may change their decision towards the private insurance, but nevertheless implies a welfare loss.

The effects of the introduction of a private insurance alternative that is only attractive to the high income earners can be summarized as follows:

- The preferred tax rate of the high income earners is reduced
 1. If $\sigma_M > 1/2$ and/or $\mu = 1$, the realized tax rate is unchanged
 2. If $\sigma_M < 1/2$ and $\mu > 1$, the realized tax rate is reduced
 - (a) If condition (60) does not hold, the realized tax rate is t_H^* .
 - (b) If condition (60) holds, the realized tax rate is t_L^* .
- The consequences for the different groups depend on what the realized tax rate is:
 - Case 1: The situation is unchanged for the M- and L-groups. The H-group has a gain from being allowed additional insurance. Private insurance reduces the conflicts of interests in the society.
 - Case 2: The M-group suffers a welfare loss, when shifted outside the political center. The tax rate is not the cost-efficient tax rate for this group anymore. For the same degree of insurance as before the median income earners has to pay a higher price, because the level of social insurance is lower and that the private insurance has a higher cost for this group at this level of social insurance.
 - Case 2a: Both the H- and L-group have a welfare gain. The gain for the H-group comes from two sources. Firstly they are allowed an additional insurance. Secondly the tax rate is reduced to this group's cost-efficient level. The gain for the L-group comes from reducing the level of social insurance, which was higher than optimal (and still is) for this group

- Case 2b: There is a welfare gain for the L-group because the tax rate now is at it's optimal level for this group. The effect for the H-group is not clear, because the tax rate now is below the cost-efficient level

Hence private insurance under condition (54) cannot hurt the low income earners and cannot benefit the median income earners.

If the median income earners are in a minority, an introduction of a private insurance leads to a welfare loss for this group, as long as $\mu > 1$. The private insurance reduces the preferred tax rate for the high income earners to a level lower than the preferred tax rate for the middle group, which results in a realized tax rate lower than optimal for the median income earners. This may lead to a positive demand of private insurance since the degree of insurance is lower than the preferred degree

$$MRS_{N,M}(t_H^*) > P_s^M(t^*),$$

and this makes the marginal value of more insurance higher. The demand is positive if

$$MRS_{N,M}(t_H^*) > P_p$$

However, the result is nevertheless a welfare loss for the median income earner. A tax rate lower than optimal means, by definition, a welfare loss. The source of this potential loss is the potential ends-against-middle situation that may result in that the median voter falls outside this group, which is a loss of political power. It is in both the H- and L-group's interests to have a tax rate below that of t_M^* .

With large deadweight costs of taxation relative to the costs of the private insurance, the private insurance alternative may be attractive to the median income earners as well. Social insurance involves no subsidizing effects for the median income earners. Hence, a positive demand for private insurance for this group implies that the deadweight costs of taxation are making social insurance more costly than the private insurance. If this is the case, the preferred tax rate for this group is reduced. This may have consequences for the realized tax rate as well.

4.1.2 Both the high- and median-income earners have a positive optimal demand

If the price on the private insurance is such that both the H- and M-group have a positive optimal demand for additional insurance, the results are different. This means that

$$P_S^H(t_H^* = \tilde{t}_H) = P_S^M(t_M^* = \tilde{t}_M) = P_p > P_c^L(t_L^*) \quad (61)$$

The optimal tax rate for the H- and M-groups are set such that the relative price on consumption when not employed is equal in the two insurance forms, thus reducing the preferred tax rates for these two groups. We recall that $\tilde{t}_M > \tilde{t}_H$, thus making the preferred tax rate for the high income group lower than for the median income group.

Proposition 5 *The introduction of a private insurance alternative that is preferred by a majority of the population, reduces the realized tax rate below the preferred tax rate of the poor, if $\mu = 1$ or if the wage differences are sufficiently large.*

In the case of $\mu = 1$, the initial tax rate is the optimal for the low income group. The relationship between the preferred tax rates then are

$$t_L^* > t_M^* > t_H^* \quad (62)$$

The median voter is in the M-group and the realized tax rate is $t_M^* = \tilde{t}_M$, which is lower than the realized tax rate without a private alternative. A reduction makes the L-group worse off, making the degree of social insurance lower than optimal. This may affect this group's decision regarding the private insurance. A tax rate lower than optimal for this group, $t^* < t_L^*$, implies that

$$MRS_{N,L} > P_c^L(t^*)$$

This group has a positive demand if the relative marginal utility at the realized tax rate exceeds the relative marginal price on private insurance. A positive demand for private insurance limits the welfare loss for the low income earners of a reduction in the realized tax rate. The total effect nevertheless is a welfare loss, because the initial level was optimal.

If $\mu > 1$, the results are less clear. If there is a positive optimal demand for private insurance for the H- and M-group, the preferred tax rates are the cost-efficient tax rates defined in (47). The preferred tax rate for the L-group is defined from the trade-off between the costs and gains of social insurance in equation (33). Recall that this tax rate is higher the higher the degree of risk aversion, the lower the deadweight costs of taxation and the higher the wage inequality. Using the same method as above, we find that the condition for the preferred tax rate of the M-group to be lower than the preferred tax rate for the L-group is

$$P_s^M(t_L^*) > P_p \quad (63)$$

The condition for the preferred tax rate of the H-group to be lower than the preferred tax rate for the L-group is

$$P_s^H(t_L^*) > P_p \quad (64)$$

Since $w_H > w_M$, the former condition is stricter. If condition (63) holds, we have that $t_L^* > t_M^* > t_H^*$. There is a majority in favor of reducing the tax rate below that of the preferred tax rate for the low-income earners. For given values of μ, τ' and q , condition (63) is more likely to hold the greater the wage differences are.

If the private insurance is attractive to both the H- and M-group the preferred tax rates for both these groups decreases, and the realized tax rate decreases as well. What the realized tax rate is in this situation depends on the degree of risk aversion, the deadweight costs of taxation, the price on the private insurance and the wage differences. The effects of the introduction of a private insurance alternative that is attractive to both the high- and median income earners can be summarized as follows:

- The preferred tax rates of the high and medium income earners are reduced
- The realized tax rate is reduced.

1. If $\sigma_M > 1/2$ and/or condition (63) holds, the realized tax rate is $t_M^* = \tilde{t}_M$. If condition (63) holds $t_L^* > t_M^* > t_H^*$.

2. If $\sigma_M < 1/2$ and condition (63) does not hold, the realized tax rate is lower than t_M^* .

(a) If condition (64) holds, $t_M^* > t_L^* > t_H^*$, and the realized tax rate is t_L^* .

(b) If condition (64) does not hold, $t_M^* > t_H^* > t_L^*$, and the realized tax rate is t_H^* .

- The consequences for the different groups depend on what the realized tax rate is
 - Case 1: In this case the median voter is in the M-group and the realized tax rate is $t_M^* = \tilde{t}_M$, which is lower than without the private insurance. It is also lower than the preferred tax rate for the L-group. This may lead to a positive demand for private insurance for the L-group as well even though the optimal demand is zero, since the degree of insurance is lower than the preferred degree

$$MRS_{N,L}(t^*) > P_c^L(t^*),$$

and this makes the marginal value of more insurance higher. The demand is positive if

$$MRS_{N,L}(t^*) > P_p$$

In this case both the H- and M-groups are made better off. The gain for the M-group is that the deadweight costs of taxation are reduced to a level that makes the costs of insurance equal in the two different forms of insurance. The H-group has an additional gain by reducing the amount of subsidies to the L-group. The L-group may be made worse off, since the degree of social insurance is lower than optimal. However, if $\mu > 1$ the realized tax rate without a private alternative was higher than optimal for the L-group. Therefore a tax rate lower than optimal is a necessary, but not a sufficient condition for the private alternative to lead to welfare loss for the L-group. A welfare loss for the L-group is possible if the level of insurance associated with this realized tax rate involves risk in such an amount that this is more costly than the excessive costs of the realized tax rate without the private alternative. In the case of

- $\mu > 1$, the tax rate without a private alternative was higher than optimal for the L-group and involved excessive costs. The L-group may thus be made better off by the reduction in the tax rate following the introduction of the private alternative, if the reduction is not too large. In this case all groups are made better off and the introduction leads to a more efficient insurance system, reducing the deadweight costs of taxation. However the reduction in the realized tax rate may very well be so large that the L-group is made worse off, exposing this group to a great amount of risk and reducing their welfare.
- Case 2a: In this case the realized tax rate is t_L^* , if none of the groups have a majority. The L- and the H-groups have a welfare gain. The gain for the former is that the degree of insurance through the social insurance is reduced to its optimal level. The gain for the latter is both in form of a higher degree of insurance and that the more costly social insurance is reduced. The effect for the M-group is not clear. The reduction in the realized tax rate is larger than the preferred reduction for this group. If the reduction is large enough, the total effect could be a welfare loss for the M-group, but the result can just as well be a welfare gain. Even though the median income earner prefers a higher degree of social insurance, the deadweight costs of taxation is reduced and it is possible that a higher degree of consumption is attainable at a lower costs.
 - Case 2b: In this case the median voter is in the H-group, as long as $\sigma_M < 1/2$, and the realized tax rate is $t_H^* = \tilde{t}_H$. This group is made better off and the gain is partly because of a higher degree of insurance and partly because of switching from the social insurance to the less costly private insurance. The realized tax rate is closer to optimal for the L-group and the degree of insurance is closer to optimal as well. The L-group is thus made better off as well. The effect for the M-group is still not clear, but in this case the tax rate is reduced more than in the first case and it is thus more probable that the introduction of a private insurance leads to a welfare loss for the M-group.

The potential loss for the L-group has two potential sources. Firstly a lower tax rate increases the amount of risk this group is exposed to, which is costly for a risk averse

individual. Secondly, a tax rate lower than optimal may make the L-group demand private insurance, which for them involves switching from the social to the more costly private insurance. The reason for the higher cost is the fact that the L-group is being subsidized in the social insurance system, but not the private one.

4.1.3 All three groups have a positive optimal demand.

If all three groups have a positive optimal demand for private insurance, their preferred tax rate is \tilde{t}_i defined in (47). I have shown that

$$\tilde{t}_L > \tilde{t}_M > \tilde{t}_H \quad (65)$$

The median voter is in the M-group. Both the H- and M-group have a gain from the introduction of the private insurance. The effect for the L-group is not clear, but their preferred tax rate is lower with a positive optimal demand for private insurance than with a non-positive optimal demand. Hence it is less likely that the L-group has a loss in welfare in this case.

Private insurance differs from the social insurance in three aspects. Firstly, the price on consumption when not employed in terms of consumption when employed is the same for all three groups through the private insurance, while being increasing in the wage through the social insurance. Secondly, the degree of private insurance is decided individually, while the degree of social insurance is the result of a democratic process. Thirdly, the price offered through the social insurance is non-linear in the presence of deadweight costs of taxation, while the price offered through the private insurance is linear. The price on one extra unit of consumption is higher, the higher the level of this consumption if attained through the social insurance, but constant if attained through the private insurance. In the case of $\mu = 1$, there are no conflicts of interests in the absence of private insurance. An introduction of a private insurance system then introduces conflicts of interests via the first aspect. In the case of $\mu > 1$, there are conflicts of interests in the absence of private insurance. An introduction of a private insurance system may reduce or change these conflicts via both these aspects. The third aspect makes it possible to reduce the amount of deadweight costs of taxation.

5 Social mobility

In this section I examine the importance of social mobility in the heterogeneous-income model. For simplicity I only consider social mobility in the middle group and only social mobility upwards. This is represented in the model by a probability α , that members of the middle group will receive a high wage, where $0 \leq \alpha \leq 1$. I disregard time-discounting and the possibility that they can return to a lower wage. α can be viewed the probability of a better job, net of the probability of losing that job in the future discounted over time. Social mobility upward for the median income earners internalizes the interests of the high-income earners. This is only interesting if the interests differ. I start without a private alternative.

The median income earner's utility function is now:

$$v_M(C) = p \left[(1 - \alpha) u(C_E^M) + \alpha u(C_E^H) \right] + (1 - p) u(C_N) \quad (66)$$

The average wage for those employed is:

$$\bar{w} = (\sigma_H + (1 - \alpha) \sigma_M) w_H + \alpha \sigma_M w_M + \sigma_L, \quad (67)$$

which I assume is the same average wage as in the model without social mobility and that $\bar{w} = w_M$.

Proposition 6 *Social mobility upwards for the median income earners increases this group's preferred tax rate in the absence of private insurance, as long as $\mu > 1$*

The first-order condition for optimal tax rate for the median voter, t_{SM}^* , is:

$$-p(1 - \alpha)u'(C_E^M)\bar{w} - p\alpha u'(C_E^H)w_H + (1 - p)u'(C_N)\frac{p}{1 - p}\bar{w}\tau'(t_{SM}^*) = 0$$

, which can be written as

$$(1 - \alpha)\frac{p}{1 - p}\frac{u'(C_E^M)}{u'(C_N)} + \alpha\frac{p}{1 - p}\frac{u'(C_E^H)}{u'(C_N)}\frac{w_H}{\bar{w}} = \frac{p}{1 - p}\tau'(t) = \frac{1}{P_s^M} \quad (68)$$

or

$$(1 - \alpha)pu'(C_E^M)\bar{w} + \alpha pu'(C_E^H)w_H = pu'(C_N)\tau'(t_{SM}^*)\bar{w} \quad (69)$$

The right-hand side of (68) is the weighed sum of the marginal rates of substitution between consumption when employed and when not employed for the median and high income earners, the weight being the probability for each income-state, with an additional weight equal to the wage differences, w_H/\bar{w} , on the high income state. In this equation the benefits are measured in terms of consumption when not employed.

I have shown that if $\mu = 1$, the preferred tax rate is identical for all three groups in the absence of private insurance. Hence social mobility makes no difference regarding the preferred tax rate without a private insurance. If $\mu > 1$, the preferred tax rates are increasing in the wage. We see that if $\alpha = 0$, (68) is identical to (33) for $i = M$ and $t_{SM}^* = t_M^*$. If $\alpha = 1$, (68) is identical to (33) for $i = H$ and $t_{SM}^* = t_H^*$. Thus, in the intermediate case where $0 < \alpha < 1$, $t_M^* < t_{SM}^* < t_H^*$. Social mobility upward for the median income earner makes the preferred tax rate higher and closer to the preferred tax rate for the high-income earners. t_{SM}^* is closer to t_H^* , the higher α and w_H/\bar{w} is. A higher tax rate implies that the preferred degree of insurance for the median income earner, C_N/C_E^M , is higher. Social mobility upward increases expected income. When insurance is a normal good (which is implied by $\mu > 1$), this increases demand for insurance and makes the preferred tax rate higher.

We show this point in Figure 3, with the costs and benefits curves of the tax rate.

The left-hand-side of (69) is the weighed sum of the high- and medium-wage costs curves. The right-hand side is as before the benefit-curve. I have previously shown that $u'(C_E^M)\bar{w} > u'(C_E^H)w_H$ for all t , with $\mu > 1$. This implies that

$$pu'(C_E^H)w_H < (1 - \alpha)pu'(C_E^M)\bar{w} + \alpha pu'(C_E^H)w_H < pu'(C_E^M)\bar{w}$$

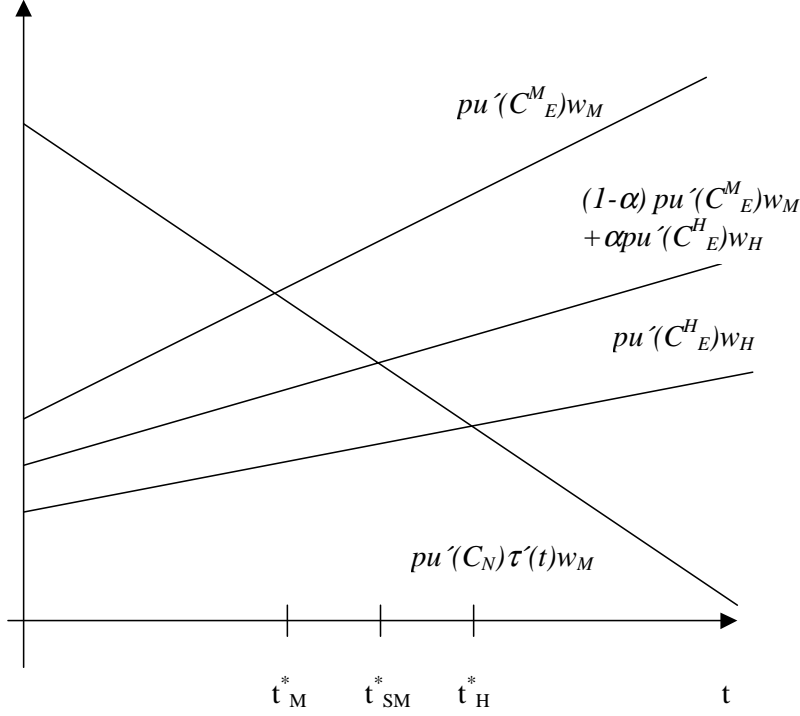


Figure 3: Social mobility, $\mu > 1$

A high-wage earner with $\mu > 1$ prefers a higher tax rate than a medium-wage earner, $t_{SM}^* > t_M^*$. A medium-wage earner with social mobility upwards internalizes these interests and thus prefers a higher tax rate than without social mobility. How higher the preferred tax rate will be depends on α , the degree of social mobility, the wage differences w_H/\bar{w} , the degree of relative risk aversion μ , and the cost of taxation, $\tau'(t)$.

The preferred tax rate is higher

- the higher α
- the higher w_H/\bar{w}
- the higher μ
- the closer $\tau'(t)$ is to one, i.e. the lower the deadweight costs of taxation

Social mobility leads to a higher tax rate. The low-wage earners are thus made worse off, since they are the ones who prefer the lowest tax rate. Social mobility upwards for the median income earner weakens the conflict of interests between the medium- and high-wage earners, but strengthens the conflict of interests between the two well-off groups and the low-wage earners.

5.1 Introduction of a private insurance alternative

What happens if we get a private alternative in this case? The previous section showed that the marginal benefit of consumption when not employed is higher for a median income earner with social mobility than without. This affects the conditions for a positive optimal demand for private insurance. Remember from section 4.1.1 that assumption (54) implied a non-positive optimal demand for private insurance and no change in the preferred tax rate for the median income earner. With social mobility, (54) is no longer a sufficient condition. A high-wage earner can benefit by buying private insurance and reduce the amount of social insurance by lowering the tax rate. This is taken into account by the median income earner if he has a positive probability of ending up as a high-income earner. Recall that with a private insurance consumption is given by

$$C_E^i = (1 - t)w_i - qQ_i$$

when employed and

$$C_N = \frac{p}{1 - p}\tau(t) + (1 - q)Q_i$$

when not employed. The public and private insurance decision can either be taken simultaneous or in two steps. I assume that the private insurance decision is more flexible than the public one. That is, I assume that the consumer is able to adapt the private insurance to his realized income level, but that the tax rate is more rigid and that the decision regarding the preferred tax rate is taken prior to an eventual increase in income. This assumption simplifies the analysis and seems reasonable.

We find the optimal solution by maximizing (66) with respect to t and Q_i . The first order conditions are

$$\begin{aligned} \frac{\partial v_M}{\partial t} &= 0 \\ \Leftrightarrow -p(1 - \alpha)u'(C_E^M)\bar{w} - p\alpha u'(C_E^H)w_H + (1 - p)u'(C_N)\frac{p}{1 - p}\tau'(t)\bar{w} &= 0 \\ \Leftrightarrow (1 - \alpha)\frac{p}{1 - p}\frac{u'(C_E^M)}{u'(C_N)} + \alpha\frac{p}{1 - p}\frac{u'(C_E^H)}{u'(C_N)}\frac{w_H}{\bar{w}} &= \frac{p}{1 - p}\tau'(t) = \frac{1}{P_S^M} \end{aligned} \quad (70)$$

and

$$\frac{\partial v_M}{\partial Q_i} = 0$$

$$\begin{aligned} &\Leftrightarrow -p(1-\alpha)u'(C_E^M)q - p\alpha u'(C_E^H)q + (1-p)u'(C_N)(1-q) = 0 \\ &\Leftrightarrow (1-\alpha)\frac{p}{1-p}\frac{u'(C_E^M)}{u'(C_N)} + \alpha\frac{p}{1-p}\frac{u'(C_E^H)}{u'(C_N)} = \frac{1-q}{q} = \frac{1}{P_p} \end{aligned} \quad (71)$$

The private insurance can be adapted to the realized income. A positive demand for private insurance is a necessary condition for a viable insurance market. This demand is given by (10) for $i = H$ or equivalently

$$\frac{p}{1-p}\frac{u'(C_E^H)}{u'(C_N)} = \frac{1}{P_p}$$

This inserted in (70) and (71) gives, from (70)

$$(1-\alpha)\frac{p}{1-p}\frac{u'(C_E^M)}{u'(C_N)} = \frac{1}{P_s^M} - \alpha\frac{1}{P_p}\frac{w_H}{\bar{w}} = \frac{P_p - \alpha P_s^H}{P_s^M P_p}$$

$$MRS_{N,M} = (1-\alpha)\frac{P_p}{P_p - \alpha P_s^H}P_s^M \equiv K(t)P_s^M(t) \equiv P_s^{SM}(t), \quad (72)$$

where

$$K(t) \equiv (1-\alpha)\frac{P_p}{P_p - \alpha P_s^H},$$

and from (71)

$$(1-\alpha)\frac{p}{1-p}\frac{u'(C_E^M)}{u'(C_N)} = (1-\alpha)\frac{1}{P_p}$$

$$MRS_{N,M} = P_p \quad (73)$$

If optimal demand for private insurance is positive with a median income, optimal consumption is given by (73). If optimal demand for private insurance is zero, the pre-

ferred tax rate is given by (73). The left-hand side of (73) is the transformation frontier between consumption when not employed and consumption when employed for the median income earner, with social mobility partially internalizing the interests of the high income earners. $P_s^{\text{SM}}(t)$ is increasing in the degree of social mobility, α , and the wage differences, w_H/\bar{w} . P_s^{SM} is increasing in t in the presence of deadweight costs of taxation, and a constant without deadweight costs.

Proposition 7 *The efficiency loss (gain) for the high income earners from increasing the tax rate is internalized by the median income earner with social mobility, making the cost of increased social insurance higher (lower).*

We see from (72) that for a given t , $P_s^{\text{SM}} > P_s^{\text{M}}$ if $K > 1$ i.e.

$$\begin{aligned} (1 - \alpha) \frac{P_p}{P_p - \alpha P_s^{\text{H}}} &> 1 \\ (1 - \alpha) P_p &> P_p - \alpha P_s^{\text{H}} \\ -\alpha P_p &> -\alpha P_s^{\text{H}} \\ P_p &< P_s^{\text{H}} \end{aligned}$$

From the definition of \tilde{t}_H , ($P_s^{\text{H}}(\tilde{t}_H) = P_p$) it follows that

$$P_p < P_s^{\text{H}} \Rightarrow t > \tilde{t}_H$$

Increasing the degree of social insurance by increasing the tax rate above the cost-efficient tax rate for the high income earners, \tilde{t}_H , implies an efficiency loss for the high income earner. A median income earner with social mobility upwards internalizes this extra cost, thus making the cost of social insurance higher with social mobility than without. For tax rates lower than \tilde{t}_H , the argument goes the other way around, making the costs of social insurance lower with social mobility than without:

$$\begin{aligned} P_s^{\text{SM}} &> P_s^{\text{M}} \quad \text{for } t > \tilde{t}_H \\ P_s^{\text{SM}} &< P_s^{\text{M}} \quad \text{for } t < \tilde{t}_H \end{aligned}$$

We remember from section four that the preferred tax rate for the median income earner is higher than the preferred tax rate for the high income earner in the presence of

a private alternative, $t_M^* > \tilde{t}_H$. This makes the cost of social insurance at this tax rate higher with social mobility than without, $P_s^{SM}(t_M^*) > P_s^M(t_M^*)$. This implies that the preferred degree of social insurance is lower with social mobility than without.

Private insurance is strictly preferred over social insurance if it is less costly even for $\tau'(t) = 1$, (no deadweight costs or $t = 0$):

$$\begin{aligned}
P_s^{SM}(\tau'(t) = 1) &\geq P_p \\
(1 - \alpha)P_s^{SM}(\tau'(t) = 1) &= P_p - \alpha P_s^H(\tau'(t) = 1) \\
\frac{1-p}{p} \frac{(1-\alpha)P_s^{SM}(\tau'(t) = 1)}{(1-\alpha) + \alpha \frac{w_H}{\bar{w}}} &\geq \frac{q}{1-q} \\
q &\leq \frac{(1-p)[(1-\alpha)\bar{w} + \alpha w_H]}{p\bar{w} + (1-p)[(1-\alpha)\bar{w} + \alpha w_H]} > 1-p
\end{aligned} \tag{74}$$

We see that an actuarially fair premium on private insurance no longer is a necessary condition for the optimal tax rate to be zero. This means that there may exist a price on private insurance that makes both the median- and high-income earners exclusively prefer the private alternative and leaves the insurance company with a positive profit and/or covers the fixed costs, when the median income earners have social mobility upwards.

Proposition 8 *The cost efficient tax rate for the median income earner is lower with social mobility upwards than without*

The cost-efficient tax rate for a median income earner, \tilde{t}_{SM} , with social mobility upwards is the tax rate that equal the relative prices on the two insurance forms. If (74) does not hold, the tax rate that equals the two prices is given by:

$$\begin{aligned}
P_s^{SM}(\tilde{t}_{SM}) &= P_p \\
(1 - \alpha) \frac{P_p}{P_p - \alpha P_s^H(\tilde{t}_{SM})} P_s^M(\tilde{t}_{SM}) &= P_p \\
(1 - \alpha) P_s^M(\tilde{t}_{SM}) &= P_p - \alpha P_s^H(\tilde{t}_{SM}) \\
(1 - \alpha) P_s^M(\tilde{t}_{SM}) + \alpha P_s^H(\tilde{t}_{SM}) &= P_p
\end{aligned} \tag{75}$$

Since $P_s^M < P_s^H$ for a given t , it follows that

$$\tilde{t}_M > \tilde{t}_{SM} > \tilde{t}_H \tag{76}$$

The tax rate that makes private and social insurance equally costly is lower with social mobility than without, the difference being greater the greater the degree of social mobility and the wage differences w_H/\bar{w} . Private insurance is preferred over social insurance at a lower tax rate for a median income earner, with social mobility than one without.

The critical value of consumption when not employed (i.e. where more consumption is equally costly through a social or private insurance) is given by

$$\tilde{C}_N^{SM} = \frac{p}{1-p} \frac{1}{\tau'(\tilde{t}_{SM})} \bar{w}$$

The similar critical value of consumption when employed is given by

$$\tilde{C}_E^{SM} = (1 - \tilde{t}_{SM}) \bar{w}$$

This critical degree of insurance is lower than without social mobility, since $\tilde{t}_M > \tilde{t}_{SM}$, as long as $\tau'' < 0$. However, Result 7 implied that the preferred degree of social insurance was lower as well. This means that we cannot say whether social mobility increases or decreases the chances for a positive optimal demand for private insurance.

The effects of social mobility depends on the relationships among the preferred tax rates without social mobility and whether optimal demand for private insurance is positive or not. Recall from section 4.1.1, that is that only the high income earners have a positive optimal demand for private insurance; t_M^* was defined by:

$$MRS_{N,M} = P_S^M(t_M^*),$$

and that $t_M^* > t_H^*$. The effects of social mobility then depend on whether the preferred tax rate for the high income earners is higher or lower than the low- income earners. Recall the cases from 4.1.1:

- Case 2a): $t_M^* > t_H^* > t_L^*$

Increasing the tax rate implies an efficiency loss for the high income earners. The cost for the median income earner is higher with social mobility than without, $P_S^{SM} > P_S^M$. This lowers the demand for social insurance and the preferred tax rate is lower as a result. Social mobility makes the preferred tax rate for the median

income earner closer to the preferred tax rate for the high income earner and the tax rates are

$$t_{SM}^* > t_H^* > t_L^*$$

Social mobility lowers the preferred tax rate. The reduction is greatest if optimal demand for private insurance is positive, but it is higher than t_H^* in either case. If $\sigma_M < 1/2$, this does not affect the realized tax rate, and social mobility only matters to the median income earners. If $\sigma_M > 1/2$, social mobility reduces the realized tax rate and leads to a welfare gain for all three groups

- Case 2b) $t_M^* > t_L^* > t_H^*$

As in Case 2a), social mobility increases the cost of social insurance, which means that the costs of the welfare state exceeds the benefits at t_M^* . This reduces the preferred tax rate for the median income earner. The reduction is greater if optimal demand for private insurance is positive, but the never so large that the preferred tax rate is lower than t_H^* . The effect on the realized tax rate depends on how large the reduction is. There are two cases

1. $t_{SM}^* > t_L^*$

In this case the median voter is still in the L-group, as long as $\sigma_M < 1/2$, and the realized tax rate is unaffected by social mobility. If $\sigma_M > 1/2$, the realized tax rate is reduced from t_M^* to t_{SM}^* and there are welfare gains for all three groups.

2. $t_{SM}^* < t_L^*$

If this is the case, social mobility shifts the median voter from the L-group to the M-group. The realized tax rate is t_{SM}^* . This implies a welfare gain for the H-group, and a welfare loss for the L-group.

If the situation without social mobility is as in section 4.1.2, that is that both the high- and median income earners have a positive optimal demand for private insurance,

the solution is clearer. Then

$$MRS_{N,M} = P_p$$

This inserted in (70) gives

$$\begin{aligned} (1 - \alpha) \frac{1}{P_p} + \alpha \frac{1}{P_p} \frac{w_H}{\bar{w}} &= \frac{1}{P_s^M(t_{SM}^*)} \\ (1 - \alpha) P_s^M(t_{SM}^*) + \alpha P_s^H(t_{SM}^*) &= P_p \end{aligned}$$

, which is identical to (75). Hence $t_{SM}^* = \tilde{t}_{SM}$. The optimal tax rate is the tax rate that makes the expected relative price on consumption when not employed offered through the social insurance system equal to the relative price offered through the private insurance. This brings the preferred tax rate for the median income earner closer to the preferred tax rate for the high income earners, but still makes it higher for all $\alpha < 1$

$$t_M^* > t_{SM}^* > t_H^*$$

The effect social mobility has under these conditions depends on what the relationships among the preferred tax rates without social mobility is. Recall that the realized tax rate could be either one of the three preferred tax rates in section 4.1.2. Which tax rate that is the median preferred tax rate depends on whether conditions (63) and (64) hold.

- Case 1): condition (63) holds, but not condition (64). This means that $t_L^* > t_M^* > t_H^*$ without social mobility

In this case, the median voter is in the M-group with and without social mobility. Social mobility then reduces the realized tax rate, implying a welfare loss for the L-group and a welfare gain for the H-group.

- Case 2a): condition (63) does not hold, but condition (64) does. This means that $t_M^* > t_L^* > t_H^*$ without social mobility

In this case, the effect of social mobility depends on whether t_{SM}^* is higher than t_L^* or not.

1. $t_{SM}^* > t_L^*$

Social mobility has no effect on the realized tax rate, as long as $\sigma_M < 1/2$.

2. $t_{SM}^* < t_L^*$

Social mobility shifts the political center from the L-group to the M-group and thereby reduces the realized tax rate, implying a welfare loss for the L-group and a welfare gain for the H-group.

- Case 2b): neither condition (63) nor (64) holds. This means that $t_M^* > t_H^* > t_L^*$ without social mobility

If $\sigma_M < 1/2$, social mobility has no effect on the realized tax rate, which still is t_H^* .

In this case social mobility weakens the conflicts of interests in the population.

If positive private insurance is optimal for all three groups, as in section 4.1.3, $t_i^* = \tilde{t}_i$ for $i = L, M, H$. Section 4.1.3 showed that $t_L^* > t_M^* > t_H^*$. Social mobility then has the same effect as in Case 1) from section 4.1.2 above.

6 Conclusion

I look at the political support for the welfare state with and without a private insurance alternative. Social insurance supplied by the government has redistributive aspects. Private insurance has not. An introduction of a optional private insurance alternative may reduce the conflicts of interests and improve overall efficiency in the economy. However the introduction may reduce the political support for the welfare state. The scope of the welfare state may be reduced as a consequence and this may be harmful for some groups. An introduction of a private alternative that reduces the scope of a welfare state through a loss of legitimacy, creates a conflict between efficiency and legitimacy, if the welfare state has a redistributive aspect.

High-income earners are paying a higher price on insurance through the social system than the median- and low-income earners. Everybody pays the same price through the private system. Private insurance thus has a relatively lower price the higher the income. If private insurance is attractive as a supplement, it is attractive as an alternative as well. Hence, all groups who prefers to have a private insurance, prefers a lower tax rate after the introduction of a private supplement. This may reduce the realized tax rate as well, if there is a majority in favor of the reduction. The preferred mix between private and public insurance hence implies a lower share of public insurance the higher the income. A reduction in the level of government spending may lead to a welfare gain for some groups of the population, but may also lead to a welfare loss for other groups. If there are some groups that experience a loss, the introduction of private insurance does not lead to a Pareto-improvement. There is a social trade-off. The welfare effect on the society as a whole depends on how high we value equality and individual freedom.

A way to counteract this effect is to make the welfare policies more universalistic and thereby making the political support for the welfare state stronger, as seen in Casamatta et al. (1998), (1999), Moene and Wallerstein (1999) and (2001). Universalistic welfare policies are often criticized for being inefficient. Designing welfare policies in an inefficient manner to secure the political support for the welfare state then clearly illustrate the conflict between efficiency and legitimacy.

References

- [1] Arrow, K. J. 1965 "*Aspects of The Theory of Risk-Bearing*" Helsinki.
- [2] Atkinson, A. B. 1999. "*The Economic Consequences of Rolling Back the Welfare State*". Cambridge: MIT Press.
- [3] Blomquist, S. and V. Christiansen 1995 "Public Provision of Private Goods as a Redistributive Device in an Optimum Income tax Model" *Scandinavian Journal of Economics* no. 97: 547-567.
- [4] Casamatta G, H. Cremer and P. Pestieau 1998 "On the political sustainability of redistributive social insurance systems" Mimeo, University of Toulouse.
- [5] Casamatta G, H. Cremer and P. Pestieau 1999 "The political economy of social security" Mimeo, University of Toulouse.
- [6] Drèze, J.H. and E. Malinvaud. 1994. "Growth and Employment: The scope for a European Initiative." *European Economy* no. 1: 77-106
- [7] Feldstein, M. S.. 1974. "Social Security, Induced Retirement and Aggregate Capital Accumulation." *Journal of Political Economy* 82: 905-926
- [8] Lindbeck, A., P. Molander, T. Persson, O. Petersson, A. Sandmo, B. Swedenborg, and N. Thygesen. 1994. "*Turning Sweden Around*." Cambridge: MIT Press
- [9] Maddison, A. 1984. "Origins and Impact of the Welfare State, 1883-1983." *Banco Nazionale del Lavoro Quarterly Review*. March: 55-87
- [10] Meltzer, A. H. and S. F. Richard. 1981 "A Rational Theory of the Size of the Government." *Journal of Political Economy*, 89: 914-927
- [11] Moene, K. O., and M. Wallerstein. 1999. "Inequality, Social Insurance and Redistribution." Memorandum No 22. 1999. University of Oslo. Department of Economics
- [12] Moene, K. O., and M. Wallerstein 2001 "Targeting and political support for welfare spending" *Economics of Governance* 2: 3-24
- [13] Roberts, K. W. S. 1977. "Voting over Income Tax Schedules." *Journal of Public Economics*, 8: 329-240

- [14] Romer, T. 1975. "Individual Welfare, Majority Voting, and the Properties of a Linear Income Tax." *Journal of Public Economics*, 14: 163-185.
- [15] Sandmo, A. 1995. "Social Security and Economic Growth." International Colloquy on "The Nordic Model of Social Security in a European Perspective." In *European Institute of Social Security Yearbook 1994*. Leuven. Acco.